

Assignment 1.

This homework is due *Thursday*, September 6.

There are total 33 points in this assignment. 30 points is considered 100%. If you go over 30 points, you will get over 100% for this homework and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper and *credit your collaborators*. Your solutions should contain full proofs. Bare answers will not earn you much.

This assignment covers sections 1.1–1.2 in Bartle–Sherbert. (Until I get the new edition, exercise numbers refer to 3rd edition.)

- (1) (Exercise 1.1.3 in textbook) Prove the Distributive laws:
 - (a) [3pt] $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$,
 - (b) [3pt] $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

- (2) (1.1.5) For each $n \in \mathbb{N}$, let $A_n = \{(n+1)k \mid k \in \mathbb{N}\}$.
 - (a) [2pt] What is $A_2 \cap A_3$?
 - (b) [3pt] Determine sets $\bigcup_{n=1}^{\infty} A_n$, $\bigcap_{n=1}^{\infty} A_n$.

- (3) Let $f : A \rightarrow B$ and $E, F \subseteq A$.
 - (a) [3pt] (Part of 1.12) Show that $f(E \cap F) \subseteq f(E) \cap f(F)$.
 - (b) [2pt] Show that not always $f(E \cap F) = f(E) \cap f(F)$. (*Hint*: as a counter-example, you can pick E and F that do not intersect *at all*.)
 - (c) [2pt] Show that not always $f(E \setminus F) \subseteq f(E) \setminus f(F)$. (*Hint*: as a counter-example, you can pick $f(E)$ and $f(F)$ that *coincide*.)
 - (d) [3pt] (Part of 1.13) Suppose, additionally, $G, H \subseteq B$. Prove that $f^{-1}(G \cup H) = f^{-1}(G) \cup f^{-1}(H)$.

- (4) [4pt] Let $f : A \rightarrow B$ and $g : B \rightarrow C$. Give “Cartesian product subset” (see def. 1.1.6 in textbook or def. of a function in lectures) definition of $g \circ f$: $g \circ f : A \rightarrow C$ is the following subset of $A \times C$: ...

- (5) (1.1.20+) Let $f : A \rightarrow B$ and $g : B \rightarrow C$.
 - (a) [2pt] Show that if $g \circ f$ is injective, then f is injective. Give example that shows that g need not be injective.
 - (b) [2pt] Show that if $g \circ f$ is surjective, then g is surjective. Give example that shows that f need not be surjective.

- (6) [4pt] (Paragraph 1.2.4g) Find a mistake in the following (erroneous!) arguments:

Claim: If $n \in \mathbb{N}$ and if the maximum of the natural numbers p, q is n , then $p = q$.

“Proof.” Proof by induction in n . Evidently, for $n = 1$ claim is true since in such case, $p = 1$ and $q = 1$.

Suppose, the claim holds for some $n \in \mathbb{N}$. Prove that then it also holds for $n + 1$. Suppose maximum of p and q is $n + 1$. Then maximum of $p - 1$ and $q - 1$ is $(n + 1) - 1 = n$. By induction hypothesis, $p - 1 = q - 1$, therefore $p = q$. Thus, the claim holds for $n + 1$ and, by induction principle, for all natural numbers.